Downside Risk-Adjusted Performance Measurement

- **Paul D. Kaplan, Ph.D., CFA**
  Chief Investment Officer
  Morningstar Associates, LLC

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Agenda

- Omega, Sharpe-Omega, & the Sortino Ratio
- Kappa as a generalization
- The Lottery Test
- Skewness & kurtosis
- Johnson distributions
- Estimating Kappa
- Impact of Kappa variant choice on performance rankings
- Summary

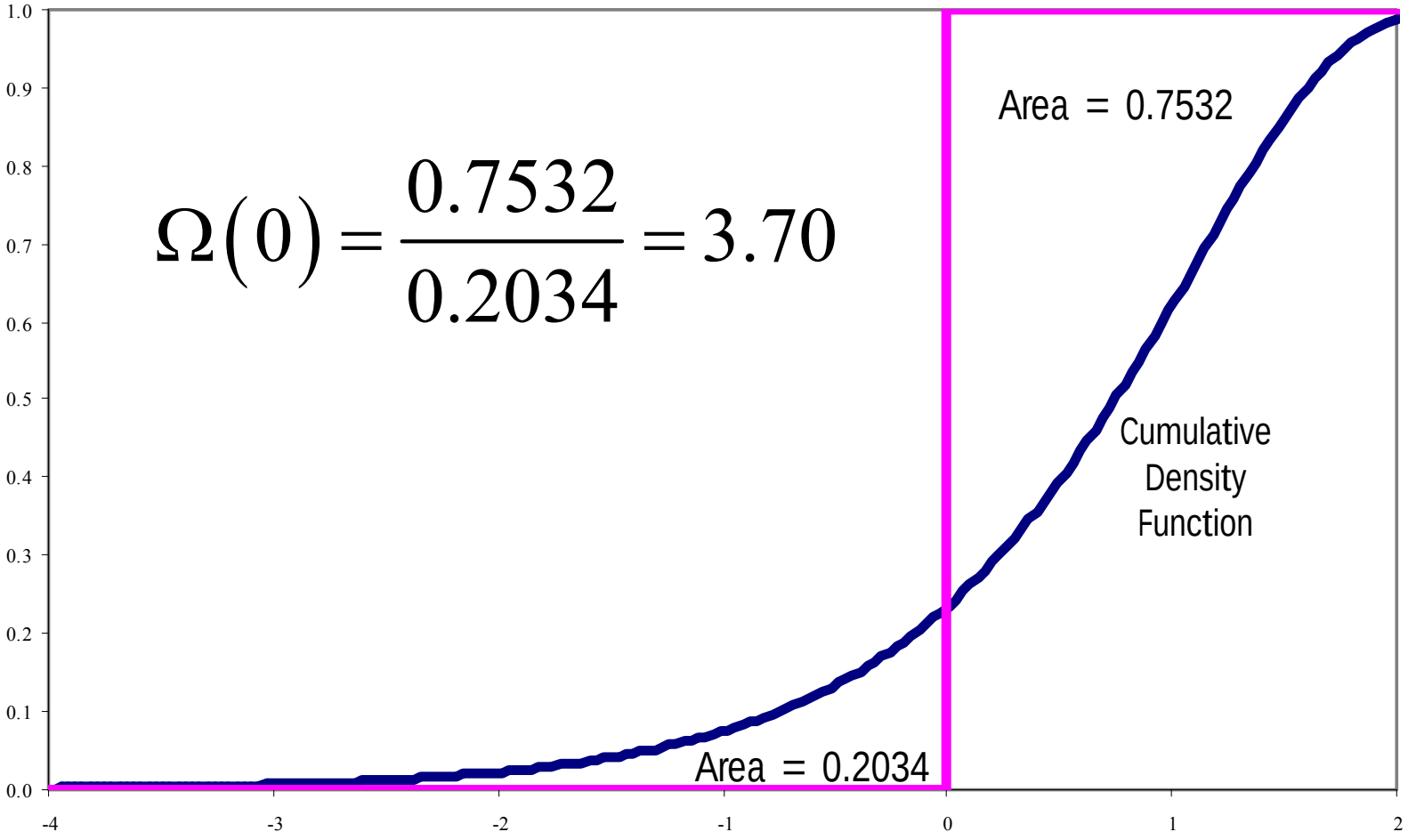
Shadwick & Keating's Omega

$$\Omega(\tau) = \frac{\int_{\tau}^{\infty} \left[1 - F(R)\right] dR}{\int_{-\infty}^{\tau} F(R)\, dR}$$

τ = threshold return

F(.) = cumulative density function of returns

Example of Omega



Kazemi, Schneeweis, & Gupta's Sharpe-Omega

$$\text{Target-Based Sharpe Ratio} = \frac{\mu - \tau}{\sigma}$$

$$\text{Sharpe-Omega} = \frac{\mu - \tau}{P(1+\tau)}$$

μ = mean of return distribution
σ = standard deviation of return distribution
$P(1+\tau)$ = price of put option (with strike price = $1+\tau$)

Equivalence of Omega & Sharpe-Omega

$$\text{Sharpe-Omega} = e^{r_f}\left[\Omega(\tau) - 1\right]$$

r_f = continuous risk-free rate

Sortino Ratio

$$S(\tau) = \frac{\mu - \tau}{\sqrt{\int_{-\infty}^{\tau} (\tau - R)^2 \, dF(R)}}$$

Lower Partial Moments (Downside Risk Measures)

$$\mathrm{LPM}_n(\tau) = \int_{-\infty}^{\tau} (\tau - \mathrm{R})^n \, d\mathrm{F}(\mathrm{R})$$

Kappa: A Generalized Downside Risk-Adjusted Measure (Kaplan & Knowles)

$$K_n(\tau) = \frac{\mu - \tau}{\sqrt[n]{LPM_n(\tau)}}$$

Omega & the Sortino Ratio as Special Cases of Kappa

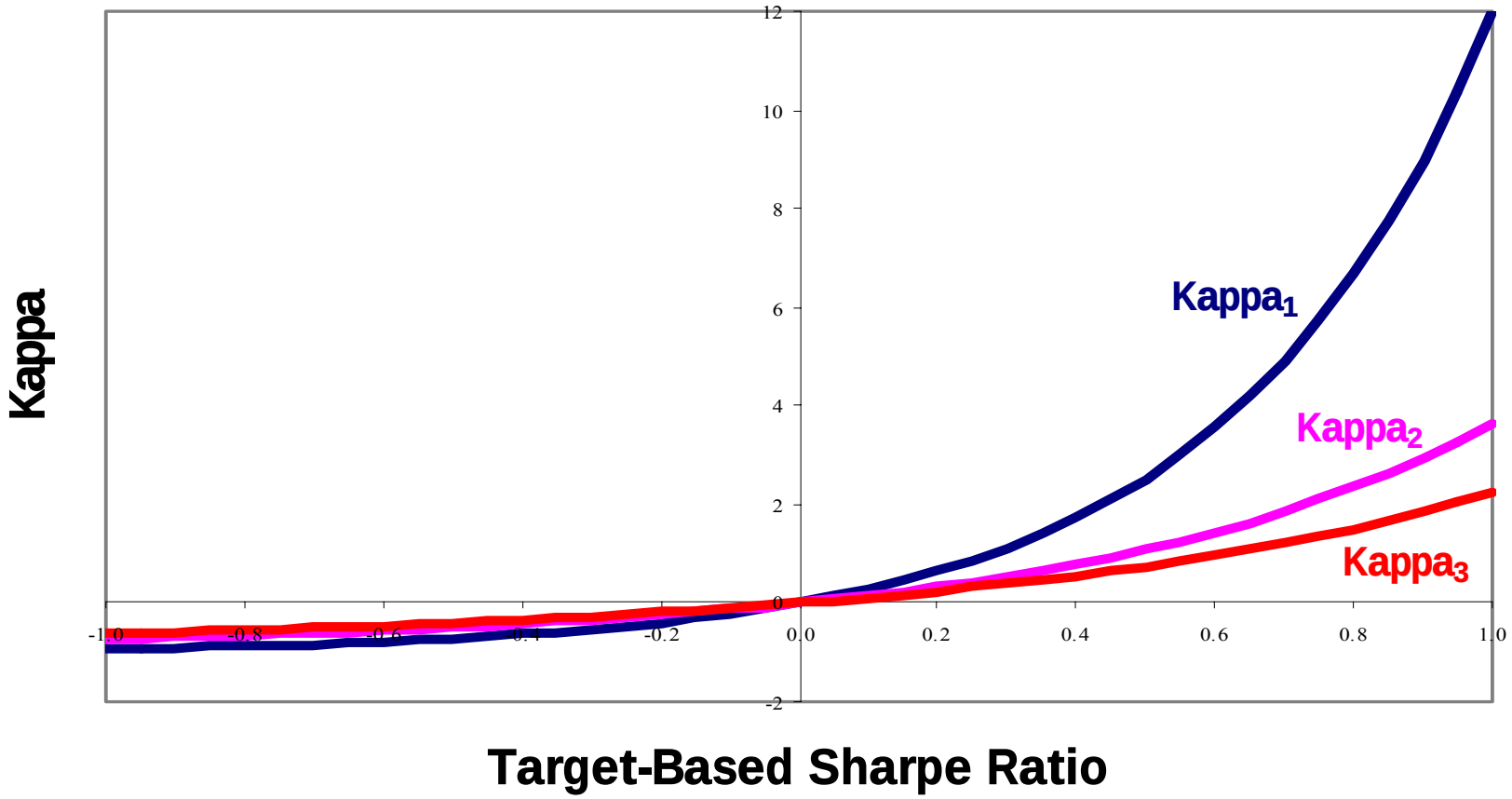
$n = 1 \Rightarrow$ Omega:

$$\Omega(\tau) = K_1(\tau) + 1$$

$n = 2 \Rightarrow$ Sortino Ratio:

$$S(\tau) = K_2(\tau)$$

Equivalence of Kappa & Target-Based Sharpe Ratio Rankings when Returns are Normally Distributed (Di Pierro & Mosevich)



Shadwick's Lottery Test

- Consider a lottery ticket
  - Price = $1
  - 1 out of 1,000,000 chance of paying $1,000,000
- Is it better to buy or sell this lottery ticket?

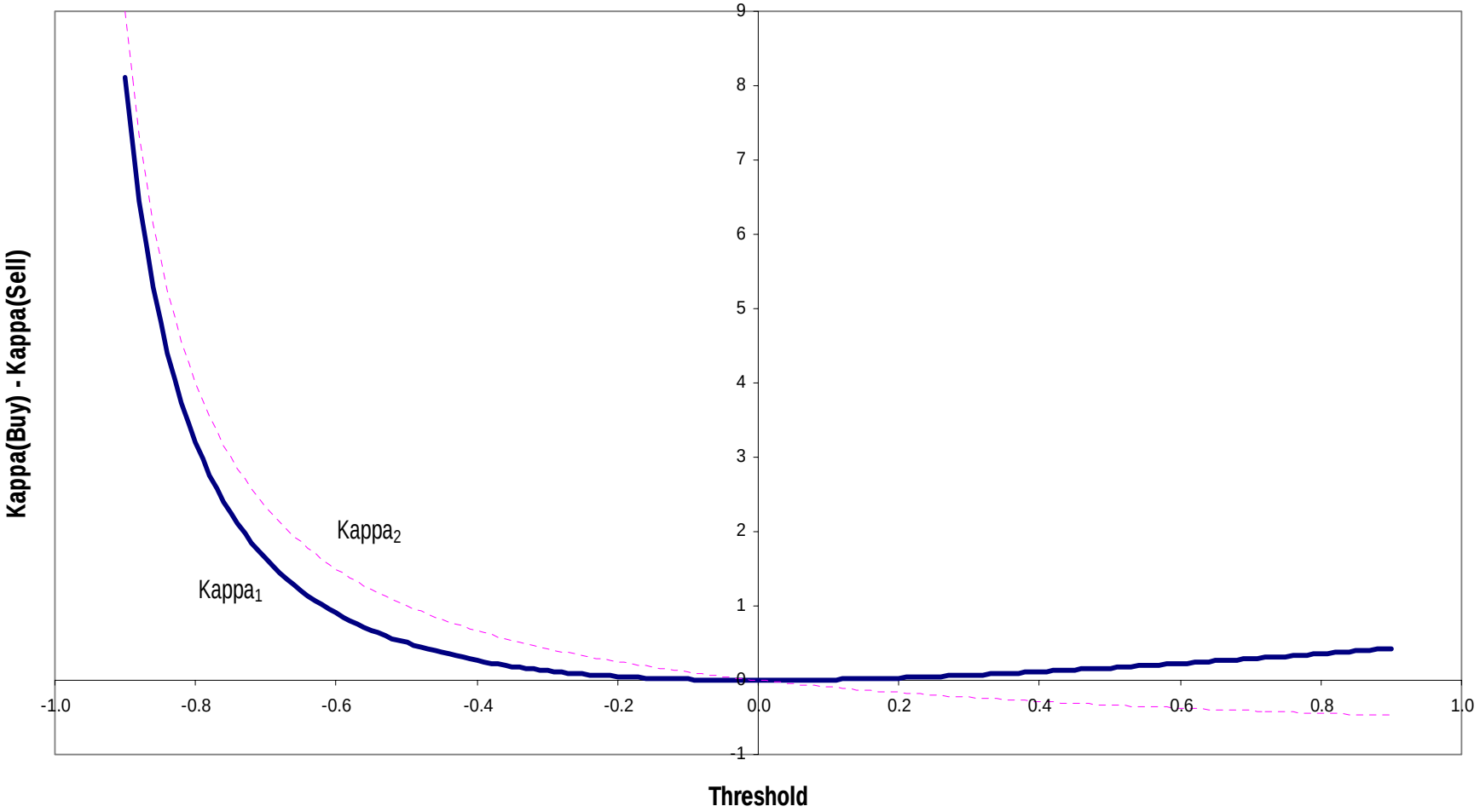
Characteristics & Ranking of Sides of Lottery Ticket

- Characteristics

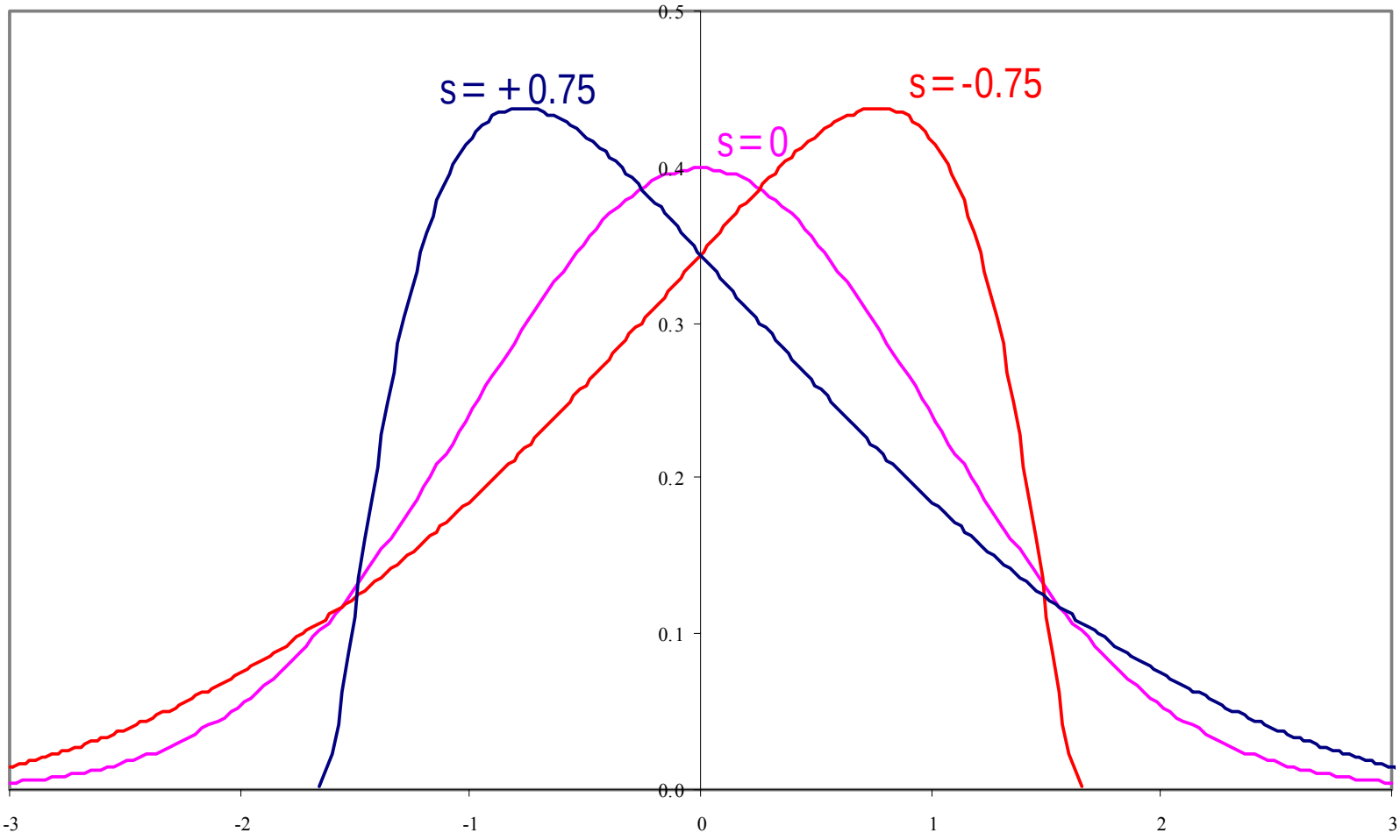
Statistic	Buy	Sell
Mean	0.0000	0.0000
Standard Deviation	999.9995	999.9995
Skewness	999.9985	–999.9985
Kurtosis	99999998.0000	99999998.0000

- Rankings
  - K_1 (Omega) ranks Buy better than Sell for all thresholds.
  - For all other values of n, ranking depends on threshold.
  - So Sortino Ratio (n = 2) fails lottery test.

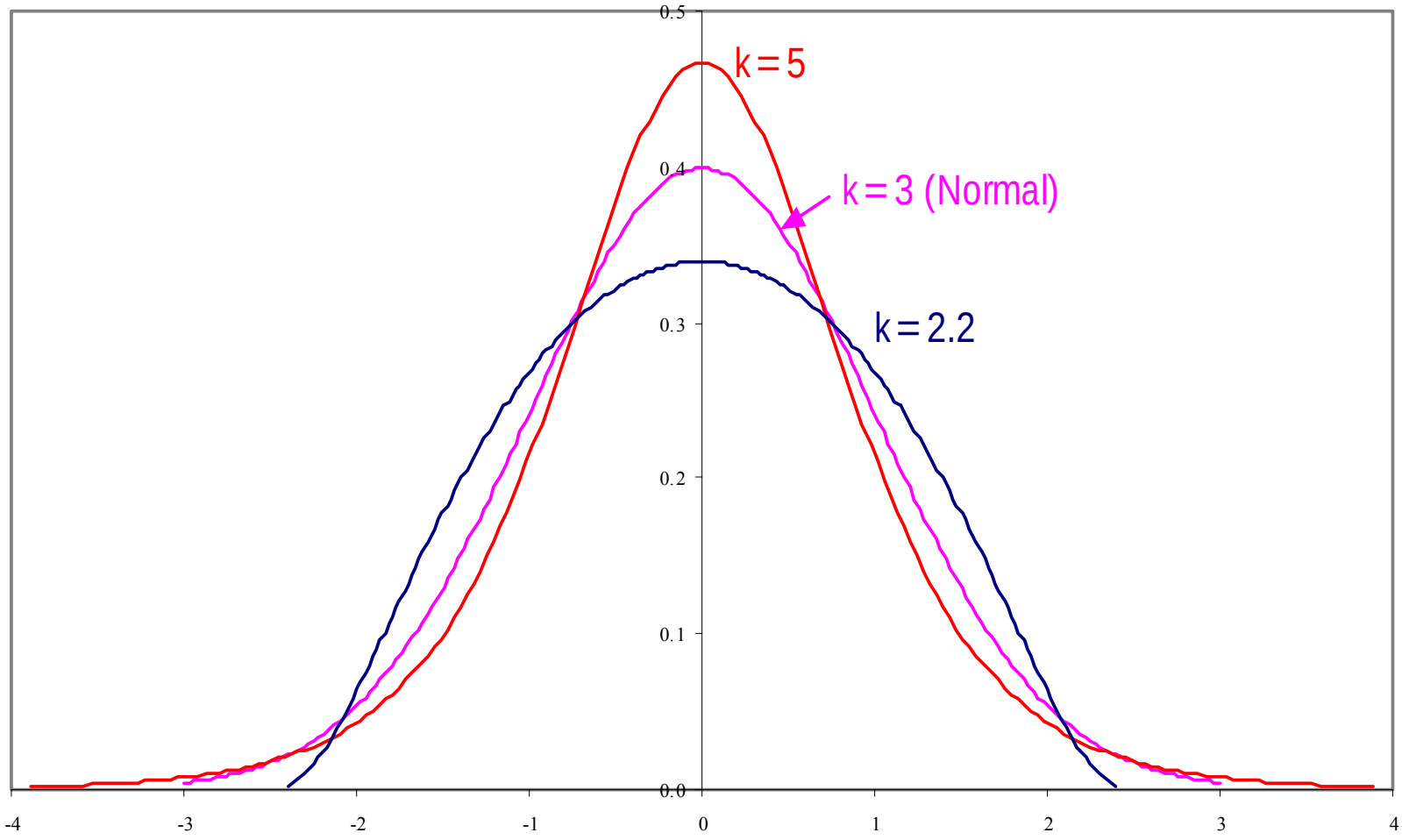
Differences in Kappa between Sides of Lottery Ticket



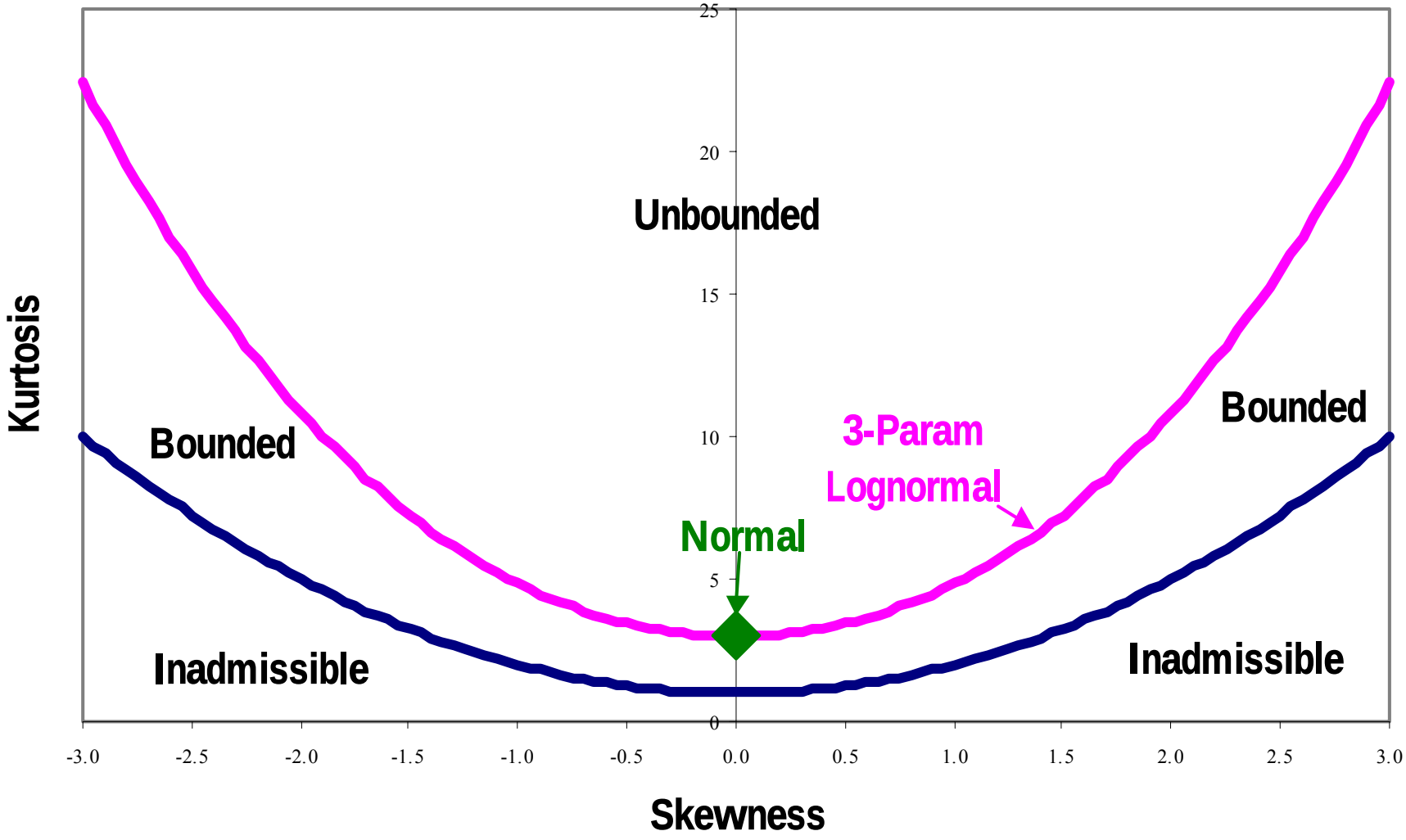
Skewness



Kurtosis



Johnson Family of Distributions

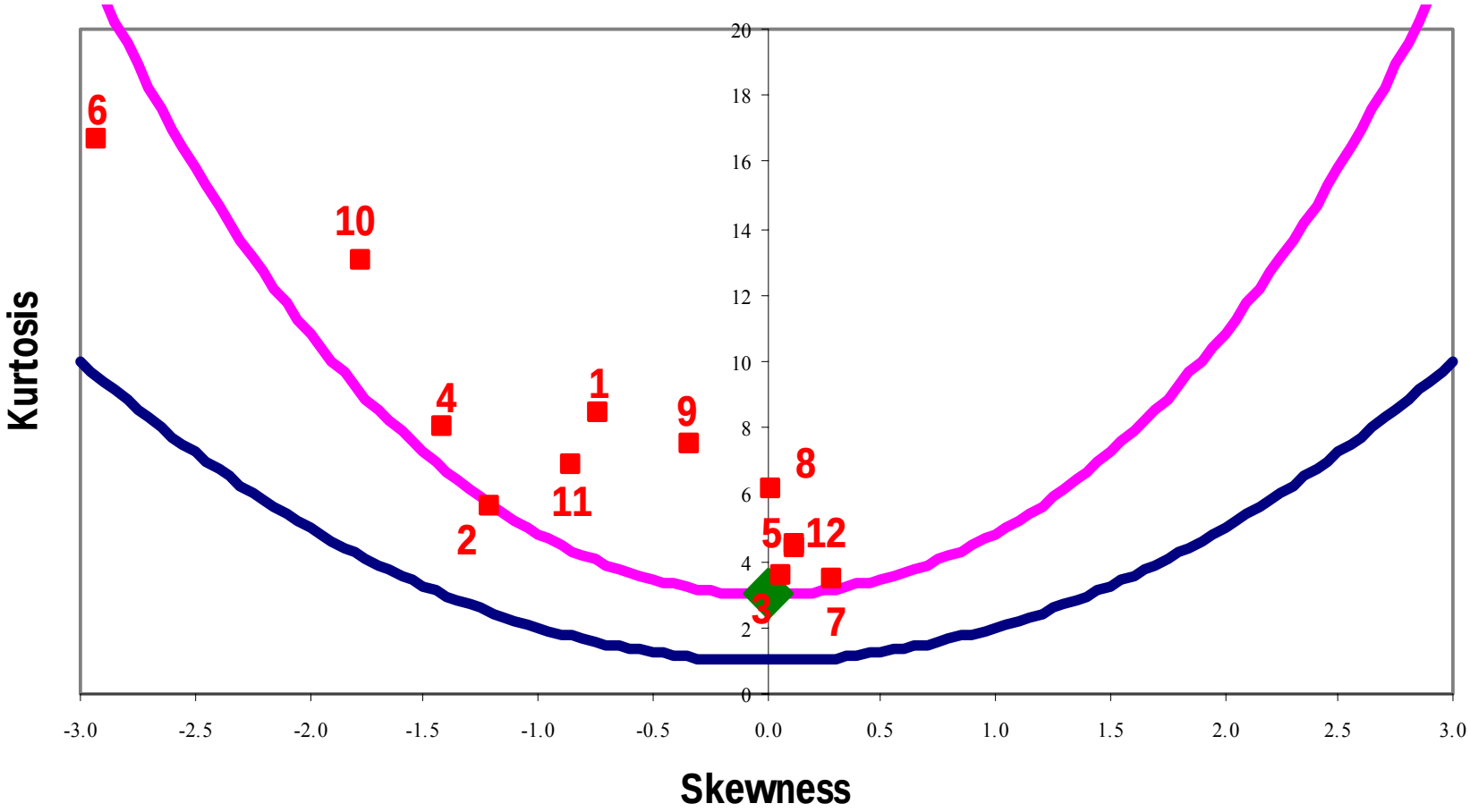


Estimating Kappa with Johnson Distributions

- Estimate mean, standard deviation, skewness, & kurtosis of returns.
- From skewness & kurtosis, select appropriate Johnson distribution:
  - $s \approx 0$ & $k \approx 3$ $\Rightarrow$ Normal
  - (s, k) near boundary $\Rightarrow$ 3-Parameter Lognormal
  - (s,k) above boundary $\Rightarrow$ Unbounded
  - (s,k) below boundary $\Rightarrow$ Bounded
- Calculate parameters of selected distribution.
- Calculate Kappa using numerical integration.

Example: HFR Monthly Hedge Fund Indices, 1/1990-12/2004

Returns in excess of T-bill



Estimates of Kappa(0) for HFR Monthly Hedge Fund Indices

Index	Johnson Dist.	$Kappa_1$	$Kappa_2$	$Kappa_3$	Sharpe
1. Distressed Securities	Unbounded	2.707	0.909	0.525	0.501
2. Convertible Arbitrage	Bounded	2.680	0.947	0.586	0.547
3. Equity Market Neutral	Unbounded	2.435	1.029	0.695	0.479
4. Event-Driven	Unbounded	2.031	0.713	0.428	0.447
5. Equity Hedge	Unbounded	1.975	0.837	0.556	0.415
6. Merger Arbitrage	Bounded	1.799	0.546	0.313	0.410
7. Macro	Unbounded	1.785	0.832	0.590	0.394
8. Sector	Unbounded	1.323	0.561	0.362	0.313
9. Fund of Funds	Unbounded	1.245	0.503	0.313	0.299
10. Fixed Income Arbitrage	Unbounded	1.110	0.401	0.235	0.282
11. Emerging Markets	Unbounded	0.798	0.334	0.213	0.223
12. Short Selling	Unbounded	-0.013	-0.007	-0.005	-0.005

Kappa-Based Ranks on HFR Monthly Hedge Fund Indices

Index	$Kappa_1$	$Kappa_2$	$Kappa_3$	Sharpe
Distressed Securities	1	3	5	2
Convertible Arbitrage	2	2	3	1
Equity Market Neutral	3	1	1	3
Event-Driven	4	6	6	4
Equity Hedge	5	4	4	5
Merger Arbitrage	6	8	9	6
Macro	7	5	2	7
Sector	8	7	7	8
Fund of Funds	9	9	8	9
Fixed Income Arbitrage	10	10	10	10
Emerging Markets	11	11	11	11
Short Selling	12	12	12	12

Summary

- Omega & the Sortino Ratio are special cases of Kappa.
- Sharpe-Omega is just a restatement of Omega.
- All Kappa variant give same rankings as the Sharpe Ratio when returns are normal.
- Omega is the only version of Kappa that passes the lottery test.
- Skewness & kurtosis can be modeled using Johnson distributions.
- Johnson distributions can be used to estimate Kappa from the first four moments.
- The 3-parameter lognormal, used by Sortino & Forsey to estimate downside risk, is a special case of the Johnson distributions.
- Choice of Kappa variant affects risk-adjusted performance rankings.

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