Shedding Light on Alternative Beta: A Volatility and Fixed Income Asset Class Comparison*

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David E. Kuenzi
Head of Risk Management and Quantitative Research

Glenwood Capital Investments, LLC
123 N. Wacker Drive, Suite 2800
Chicago, IL 60606

dkuenzi@glenwood.com

Phone (312) 881-6520
Fax (312) 881-6501

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Volatility is an alternative beta—a risk premium captured by hedge fund managers and investment bank proprietary traders—that is today moving closer to the main stream and should be thought of as a veritable asset class.[1] For many investors, it is difficult to derive intuition as to why volatility should deserve an ongoing allocation within a larger portfolio. If volatility is an asset class, then to what accepted asset class can it be compared? Why is there a risk premium over the long-term for investing in this asset class? Who is willing to pay this risk premium and why? In what environments might the risk premium be too narrow or negative and in what environments might it be substantial? These are critical questions for the institutional investors attempting to diversify various systematic exposures (or beta exposures) across a broader portfolio.

There is a strong case that a volatility investor can expect to earn positive returns over time just as a fixed income, credit, or equity investor would. To best understand this, it is helpful to compare the volatility asset class to the fixed income asset class. As such, the purpose of this paper is to explore the likenesses between volatility and fixed income in order to more firmly establish the case for volatility as an asset class.

Broad Comparison

We liken a short volatility exposure to a long bond exposure. Just as a bond issuer is willing to pay (and is in fact required to pay) a rate of interest in excess of the expected rate of inflation, a buyer of volatility is required to pay an implied volatility level in

[1] For evidence of hedge fund exposure to volatility, see Kuenzi and Shi [2007].

excess of the expected realized volatility.[2] In a general sense, the risk premium in the case of fixed income can be thought of as compensation for the uncertainty in the rate of inflation and for interest rate and duration-driven volatility. In the same way, the risk premium in the case of volatility can be thought of as compensation for uncertainty in the level of realized volatility and the volatility of implied volatility (which can lead to large mark to market moves in volatility instruments). In cases of severe and unexpected inflation, it is better to be short bonds, and in cases of severe and unexpected realized volatility, it is better to be long volatility. Market required interest rates increase dramatically in the case of severe inflation, thus driving the value of the bond investment lower, in the same sense, market levels of implied volatility respond to increases in realized volatility, thus driving the value of a short volatility position lower.

Derman [2003] draws some comparisons between bonds and options as well, noting the equivalence between the way the instruments are quoted—interest rates as parameters for bonds and volatilities as parameters for volatility instruments. The quoted interest rate must be plugged into a present value formula in order to derive the price of the bond, while the quoted implied volatility must be plugged into a Black Scholes (or related model) in order to get the price of the option. Derman [2003] and Derman, et. al. [1998] also notes the similarities between the yield to maturity and implied volatility with reference to the instruments themselves. Both are essentially summary measures,

[2] If held to expiration or maturity, a short volatility portfolio (such as a delta-hedged short straddle, a short variance swap, or a short volatility swap) generally has a payout equal to (or approximately equal to) the implied volatility at the time of initial trade execution minus the actual realized volatility of the underlying over the period the portfolio was held. As such, if implied is consistently larger than actual realized, the volatility seller will make money.

indicating the average levels (of interest rates and volatility, respectively) that make the current price of the instruments fair.

Mean reversion is another commonality, as both interest rates and volatility are understood to be mean-reverting processes. Neither volatility nor interest rates can go beneath zero; negative volatility is not defined either mathematically or conceptually, and the occurrence of negative interest rates is virtually impossible. Additionally, neither will grow perpetually through time as a stock price would.

Finally, both have instruments of varying maturities or expiries. As such, both have term structures that are related to a given level of interest rates or volatility. Given the mean-reverting quality noted above, this also means that the behaviors of their term structures have similar characteristics.

The similarities between fixed income and volatility are summarized in Exhibit 1. We explore these commonalities—as well as some notable differences—in the sections below. We also explore the various methods that investors have at their disposal for capturing the volatility risk premium.

Establishing Comparable Data Series

In the ensuing sections, we make some very concrete comparisons between the fixed income and volatility asset classes. In doing so, we need to establish parallel instruments or metrics between the two markets. We note that the yield on a 2-year Treasury bond can be thought of as the expected rate of inflation during the next 24 months plus a premium. Similarly, we note that the VIX Index,[3] the most commonly quoted volatility index, measures one-month volatility of the S&P 500 Index—or expected realized volatility over approximately 21 trading days—plus a premium. The frequency of updated inflation information is monthly (the monthly CPI report), while the frequency of updated realized volatility information is daily (daily squared return of the S&P 500 index). The monthly CPI number can be thought of as updated information on the extent to which the fixed income investor is capturing the intended risk premium. By the same token, the daily realized volatility number can be thought of as updated information on the extent to which the short volatility investor realized the intended risk premium on any particular trading day. Initial one-day implied minus one-day realized volatility can be thought of as a one-day accrual. If the annualized one-day realized is lower than the initial implied, the short vol investor will generally have a positive P&L for that day, and vice versa if the realized is higher than the original implied. Therefore, in a very broad sense, these two time series (the 2-year Treasury yield and the level of the VIX Index) present parallel items for comparison. We use monthly data for the 2-year Treasury and daily data for the VIX, as these frequencies represent the respective frequencies for the updating of information regarding the capture of the intended risk premium.

The Risk Premium

Despite the existence of environments in which it is better to be short bonds, the literature suggests that over time there is a positive risk premium to owning bonds (see, for

[3] The VIX index, calculated by the Chicago Board of Trade, is an index measuring the one-month implied volatility of S&P 500 options contracts. For the VIX calculation methodology, see CBOE (2003).

example, Ibbotson and Sinquefield [1976], Cox, Ingersoll, and Ross [1981], and Feinman [2002]).[4] By the same token, there are now some studies that suggest that there is a positive risk premium over time to being short volatility—or conversely, that there is a negative risk premium to being long volatility. Bakshi and Kapadia [2003a] find that for a long delta-hedged call position on the S&P 500 there is a negative risk premium (a positive risk premium to being short volatility). For at-the-money options, they find a risk premium of approximately -0.13% of the underlying index value and -8% of the value of the option.[5] In Bakshi and Kapadia [2003b], they note that:

> Because of the negative correlation between market index returns and market index volatility, buyers of options may be willing to pay a premium because a long position in volatility helps hedge marketwide risk. (p. 51)

In a study focused more purely on variance swaps, Carr and Wu [2004] find that "the variance risk premia are strongly negative" for the long variance investor. (p.37) Finally, in a study focusing on European equity index volatility, Hafner and Wallmeier [2006] note that:

> "…results show that on average, investors are willing to accept a heavily negative risk premium for being long in realized variance. Equivalently, investors who are sellers of variance and are providing insurance to the market, require a significantly positive risk premium." (p. 15)

In short, buyers of options and volatility products are willing to pay sellers an insurance premium. As volatility tends to spike in a difficult environment for risky asset classes, and is therefore negatively correlated with the returns to equities and credit, investors are

[4] Morningstar provides an ongoing annual update of the returns and risk premiums associated with various asset classes in a publication called *Stocks, Bonds, Bills, and Inflation Yearbook 2007* [2007].

[5] In Bakshi and Kapadia [2003b], the authors come to a similar result for options on individual stocks, except that the risk premium is lower in absolute value. The reason they give for this (in keeping with their evidence) is that the systematic component of volatility is priced but the stock-specific, or idiosyncratic component, is not.

willing to pay a premium to hold this asset. It is this premium that the investors in the volatility asset class can capture over time.

The notion of a positive risk premium for bonds is straightforward: if the bond yield is higher than realized inflation, the investor has benefited from holding the security. Panel 1 of Exhibit 2 shows the 2-year constant maturity Treasury yield versus both prior realized inflation (average inflation during the preceding 24 months) and realized inflation (average inflation during the ensuing 24 months).[6] It is clear that except in instances of sudden spikes in inflation, the risk premium has been positive for fixed income investors.

In Jackwerth and Rubinstein [1996], the authors note that "option-implied volatility is almost always biased upward from prior historical realizations." (p. 1613) This is in keeping with the notion of a positive risk premium for volatility sellers—who essentially capture this difference between initial implied volatility and trade ex-poste realized volatility. Panel 2 of Exhibit 2 shows the VIX index versus both prior realized volatility during the preceding 21 trading days and realized volatility during the ensuing 21 trading days. As is the case for fixed income, investors are generally granted a positive risk premium except in cases of extreme spikes in volatility. The VIX is higher than both prior realized and realized volatility in most cases. In both Panel 1 and Panel 2 of Exhibit 2, it is somewhat clear that realized inflation and realized volatility only exceed yields

[6] In Exhibits 2 and 3, we use the Constant Maturity 2-Year Treasury rate (Bloomberg item H15T2Y Index), as this rate will be directly comparable to the 24-month simple average of the monthly inflation rate. For volatility, we use the VIX index (Bloomberg item VIX Index). This can be thought of as a "constant maturity" one-month implied volatility level, which makes it directly comparable to the 21-day realized volatility.

and implied volatilities, respectively, when there has been a sudden and likely unforeseen spike in the respective series.

The information is Exhibit 2 is summarized in the tables in Exhibit 3. Panel 1 shows that both over the entire period and over each sub-period, the risk premium for investors in the 2-year Treasury bond was positive—based both on prior 24-month inflation as well as the inflation levels that were actually realized over the remaining life of the bond. Panel 2 of Exhibit 3 shows that the same held true for volatility investors. Implied volatility exceeded both prior realized volatility and actual realized volatility for both the entire period (from 1/2/1990 to 12/28/2006) and for both sub-periods. Together, Panel 2 of Exhibit 2 and Panel 2 of Exhibit 3 suggest that there is a fairly steady risk premium offered to investors selling volatility.

Mean Reverting Property and the Nature of the Term Structure

As noted above, it is widely accepted that both interest rates and volatility are mean-reverting processes. Literature concerning the mean reverting nature of interest rates is abundant and includes, among many other sources, Vasicek [1977] and Cox, Ingersoll, and Ross [1985]. Literature concerning the mean reverting nature of volatility includes, among other sources, Hull and White [1987] and Heston [1993].[7]

[7] For additional analysis and sources concerning the mean-reverting nature of interest rates, see Brigo and Mercurio [2001], and for additional analysis and sources concerning the mean-reverting nature of volatility see Psychoyios, Skiadopoulos, and Alexakis [2003].

A shared mean reversion characteristic leads to many similarities between the two asset classes. The most basic mean-reverting processes for each (corresponding to Vasicek [1977] and Hull and White [1987], respectively) can be written as:

$$dr_t = k_r(a_r - r_t)dt + \sigma dW_t^r \quad (1)$$

$$dV_t = k_V(a_V - V_t)dt + \sigma dW_t^V \quad (2)$$

Where k is the mean reversion parameter, a is the long-term mean, r_t is the instantaneous interest rate, V_t is the instantaneous level of implied volatility, σ is the volatility of rates and the volatility of implied volatility (for each equation, respectively), and dW_t is a standard Brownian motion. First, we note that the equations above are nearly identical; one can model the dynamics of each series using the same framework. Second, we estimate the mean reversion parameter k using both sets of data (the 2-Year Treasury Rate and the VIX). The results, shown in Exhibit 4, confirms that both are mean reverting processes with a generally similar rate of mean reversion.[8]

The fact that both the fixed income and volatility markets involve mean reverting processes and have instruments with various times to maturity provides for strikingly similar term structures. Exhibit 5 shows simple term structures for both markets. Both are upward sloping under normal circumstances. From a fixed income perspective, this reflects the increased inflation uncertainty over the long term as well as the increased instrument price volatility associated with a higher duration. From a volatility

[8] We estimate the mean reversion parameter k using time series regression. We note that such a regression has the form $V_{t+dt} = \alpha + \phi V_t + \varepsilon$. We subtract V_t from both sides, set $k = -(\phi - 1)$, $a = \alpha / k$ and $\varepsilon_t \approx \sigma dW_t$, and we get equation (2). A similar process reproduces equation (1). As such, we estimate the mean reversion parameter k from the time series regression and report the related t-statistic for ϕ.

perspective, this reflects the increased uncertainty as to what realized volatility might be over the longer term as well as the increased instrument price volatility associated with higher vega.[9]

It is also informative to consider the term structure behavior of the two markets. Given that both markets have mean reversion characteristics, one finds that the longer end of the curve is less volatile than the shorter end of the curve. This is driven by an expectation among market participants that some degree of mean reversion is likely to occur by the time the latter dates are reached. The near term maturities / expiries, on the other hand, are much more likely to be affected by temporary shocks. Exhibit 6 shows interest rate volatility and the volatility of implied volatility at different points of the term structure. Both the fixed income time series and the volatility time series have the same pattern—higher volatility for shorter maturities and lower volatility for latter maturities.

Another characteristic of these term structures is that both tend to invert when the absolute level of interest rates and volatility, respectively, is very high. This again is driven by the notion of mean reversion. If short-term interest rates increase to historically high levels, market participants are likely expect a reversion toward the long-term mean within a few years. As such, long-term interest rates are unlikely to rise as much. The same holds true for volatility. When short-dated volatility spikes, market participants are likely to take the view that markets (and therefore implied volatility) will settle down over the longer term. This will lead to a muted reaction of longer term

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[9] An option's vega is defined as $\partial \quad \partial V_t$, or the change in the option's price for a change in the level of implied volatility. Vega is increasing in time to expiry.

implied volatility to extreme events. This can best be seen in Exhibit 7, which shows the relationship between the slope of the term structure and market level. As the level of both interest rates and implied volatility increase, the term structure of both interest rates and volatility are likely to invert.

Overall, the mean reversion and term structure related similarities between the two asset classes are dramatic. As such, many of the same types of analytics are pertinent in the analysis of these markets and in determining the ideal time to be long the related risk premium.

Some Differences

There are, however, some key differences between fixed income and volatility investing. The most critical of these is exposure to a market liquidity event or market sell-off. High-quality bonds such as U.S. Treasuries will tend to perform well in a financial crisis, whereas a short volatility position will almost surely experience significant negative returns in a market crisis. The Treasury bond investor is long liquidity crisis risk; the short volatility investor will generally be short liquidity crisis risk. Treasuries are, however, highly exposed to any potential inflation scare. While volatility is not directly exposed to such an event, short volatility positions would likely underperform in an inflation scare as well. Overall, short volatility positions are likely to underperform in any destabilizing environment.

This has a number of implications for the means by which this alternative beta might be captured and the ways that such a volatility portfolio might be included as an element of a broader investment program. Volatility exposures must be modeled with extreme events in mind, as this is what the short volatility investor is selling insurance against. The investor's volatility exposure should be such that the portfolio will remain in good health in the case that an extreme event were to occur. Ideally it would be positioned such that the investor could sell yet more volatility at wider levels in such a situation. This also points to the skill brought to bear by the investment manager. Knowing how much volatility to sell and at which expiries is a complex undertaking.

The differences between the two markets compounds as one considers the minutia. Options, for instance, have volatility smiles—an interaction between the moneyness of an option and the associated implied volatility. Bonds have coupons and an interaction between coupon level and yield to maturity. Bonds also have a sense of on-the-run (the most recently issued and most heavily traded) versus off-the-run. If we consider callable bonds, the credit quality of various issues, and other characteristics of various types of bonds, the similarities and differences become extraordinarily complex.

Capturing the Risk Premium

In order to gain exposure to this alternative beta—to capture the risk premium associated with the volatility asset class—investors employ a variety of strategies. These include covered call writing, the sale of puts against short positions, the sale and delta-hedging of options, the sale of variance swaps and volatility swaps, and the sale of other more exotic variance swaps such as corridor variance swaps, conditional variance swaps, and gamma swaps. Exhibit 8 provides an overview of the various methods that investors employ in order to obtain beta exposure to the volatility asset class.

The simplest of these is probably covered call writing—selling options on securities or futures for which the investor is long the underlying. This is a well-accepted strategy for which an index and much information is available.[10] The sale of puts against short positions is the analogue to covered call writing for the short seller. Just as the covered call writer's gains are stymied in a stock market rally, the gains of a seller of covered puts are reduced in a severe stock-market sell-off. In both cases, the seller of volatility should experience excess risk-adjusted returns over time due to the existence of the volatility risk premium combined with the diversification benefits of adding this alternative volatility beta to the exposures inherent in a traditional portfolio. The unattractive aspect of these strategies is that they alter the payoff profile and return distribution of the underlying portfolio.

Delta-hedged option trading is the traditional method for obtaining volatility exposure without incurring directional exposures. This involves the sale of typically short-dated at-the-money options and the ongoing hedging of the directional exposure of the options. If, for instance, the investor sells eight 0.50-delta call options representing 800 index units, then the investor will buy futures or ETFs representing 400 index units. If the stock moves higher so that the delta increases to 0.75, then the investor will purchase an

[10] This has been one of the more popular strategies during the last few years. So much so, that the CBOE has launched a covered call index (see Whaley [2002] and Feldman and Roy [2005]).

additional 200 index units. In this way, the investor remains neutral to all pure directional exposures but long volatility. One problem with this approach is that it is operationally and systems intensive, requiring ongoing adjustments to the hedge.[11]

Variance swaps and volatility swaps require no such ongoing adjustments. The simplicity of these instruments from the client's perspective makes it quite easy to get access to volatility beta. The payoff of a variance swap is:

$$\begin{aligned} Payoff_{VarSwap} &= N\left(\quad \sum \left(\quad \right) - \quad \right) \\ &= N\left(\quad \sum \quad \right) \end{aligned} \tag{3}$$

where M is the total number of monitoring periods (usually business days) between swap inception at time t_0 to swap maturity at time T. S_i is the price of the underlying on day i, R_i is return of the underlying on day i, K_{var} is the initially agreed upon variance strike expressed in volatility points squared, and N is the notional amount. The investor has simply to enter into this contract and volatility exposure is locked in for the life of the swap, with no other hedging activity required.[12] (Volatility swaps are similar, except that the payoff is to realized volatility minus a volatility strike. Variance swaps are easier for dealers to hedge and thus tend to be more liquid.) In this regard, variance and volatility swaps have opened up the volatility asset class to a whole new set of investors by providing the means for user-friendly exposure to volatility beta.

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[11] Derman, et. al. [1998] make the same observation. They also note the simplicity of obtaining volatility exposure through the use of "realized volatility contracts" and juxtapose this very simple approach with the more complex delta-hedging approach. (This brief piece was clearly well ahead of its time.)

[12] Investors should be wary of the varying levels of vega exposure ($\partial P / \partial \sigma$), of the change in position value for a change in implied volatility) that the variance swaps are subject to over the course of their existence and the associated mark-to-market implications. See Kuenzi [2005] for details.

If a short variance swap can do the trick, why not also throw in some short VIX futures positions? VIX futures are ideal instruments for the purposes of expressing views on the direction of implied volatility, but they do not allow investors to capture the risk premium as described here. If held to expiry, VIX futures provide a payoff roughly equal to the level of the futures contract at the time of sale (the VIX expected at expiry as of the date of the sale) minus the level of the VIX at expiry. This payoff is independent of realized volatility except to the extent that realized has an impact on implied. In other words, VIX futures provide pure vega exposure (exposure to changes in implied volatility) with no gamma exposure (exposure to the difference between implied and realized).

For those wishing to express more complex views, however, their now exist a variety of more exotic variance swaps, such as corridor variance swaps, conditional variance swaps, and gamma swaps. These instruments allow investors to express views on volatility and the level of the underlying simultaneously. As such, they also allow investors to express views on the shape of the volatility skew. The payoffs of these instruments are as follows:

$$Payoff_{CorridorVarSwap} = N\left(\quad \sum \quad \right) \tag{4}$$

$$Payoff_{ConditionalVarSwap} = N\left(\quad \right)\left(\quad \sum \quad \right) \tag{5}$$

$$Payoff_{GammaSwap} = N\left(\quad \sum \left(\quad \right) \quad \right) \tag{6}$$

Where *L* is a lower bound on the underlying, *U* is an upper bound on the underlying and *D* is the number of days that the underlying has spent in the given range. *D* is defined as:

$$D = \sum_{i=1}^{M} 1_{\{S_{i-1} > L,\ S_{i-1} \le U\}} \qquad (7)$$

Each of these instruments can play a different role for investors. The corridor variance swap allows investors to obtain access to the volatility risk premium subject to the underlying being within a range. (It's important to remember that if *L* is set to zero and *U* is set to infinity, the corridor variance swap generally collapses to a standard variance swap.[13]) If an investor wanted to take advantage of the volatility risk premium but also believed that the underlying was heading straight up, the investor might sell a corridor variance swap with *L* equal to 95% of the current underlying price and *U* equal to infinity (no upper bound). In this case, realized variance will accrue so long as the underlying is above 95% of its level at swap inception.

Conditional variance swaps are quite similar to corridor variance swaps. As noted by JP Morgan [2006]: "The difference between a corridor and a conditional is that in a corridor variance realized outside the range is counted as zero, whereas in a conditional variance swap all variance realized outside the range is simply ignored." (p. 6) As such, the short corridor swap investor would prefer that the underlying immediately go outside the range and stay there for the life of the swap, while the short conditional swap investor would prefer that volatility remain very low within the range.

[13] See Carr and Lewis [2004] for a more precise treatment of this topic.

Gamma swaps simply scale realized variance by the level of the underlying. These can be useful in trading the volatility skew (long variance swaps and short gamma swaps if the skew is steep and vice versa if the skew is flat; see Mougeot [2006] for details). Overall, the instruments available to investors provide for an ability to quite easily invest in this risk premium and to express any number of nuanced views in the process.

Given the variety of methods for accessing volatility risk premia noted above, it is clear that volatility portfolio management processes can run the gamut from relatively simple to highly complex. In any case, the central decision for the volatility investor wishing to receive this risk premium is to determine the extent to which the premium is sufficient at any given time—whether the level of implied is large enough to offset potential spikes in realized. (Again, this is not too different from the fixed income investor who must decide whether the risk premium on bonds is sufficient to offset potential increases in inflation.) Variance and volatility swaps—along with the related exotics—allow investors to focus on these critical decisions rather than being overwhelmed with the analytical, trading, and operational issues associated with running delta-hedged options portfolios. Banks are offering these instruments on a variety of underlyings, and especially on those underlyings with liquid options markets. This includes a variety of equity indexes, individual equities, and currencies. This gives volatility investors a rich set of instruments to choose from. The investor is therefore left to build investment processes focused on an evaluation of the volatility risk premium associated with each of these underlyings and the best way to structure a portfolio of volatility products without a tremendous amount of operational overhead.

Conclusion

In coming to terms with volatility as an asset class, it is intuitively helpful to build a comparison between volatility and fixed income. These two asset classes share a wide variety of characteristics and from many perspectives can be approached through a similar lens. The drivers of their respective risk premia, the way they are quoted and priced, and their mean-reverting and term structure properties share many parallels. While these similarities have existed as long as options have been traded, until recently it has been difficult for investors to efficiently access these risk premia due to the analytical, trading, and operational issues involved with delta-hedging. With the advent of variance swaps (along with their more exotic cousins), it is now just as straightforward for investors to add this alternative beta to their portfolios as it is to buy government bonds. Given that the evidence shows that there is indeed a positive risk premium associated with selling volatility and that the related return stream is not 100% correlated with the returns of traditional assets, it is clear that the addition of volatility investments to a traditional portfolio is likely to increase risk-adjusted returns over the long term.

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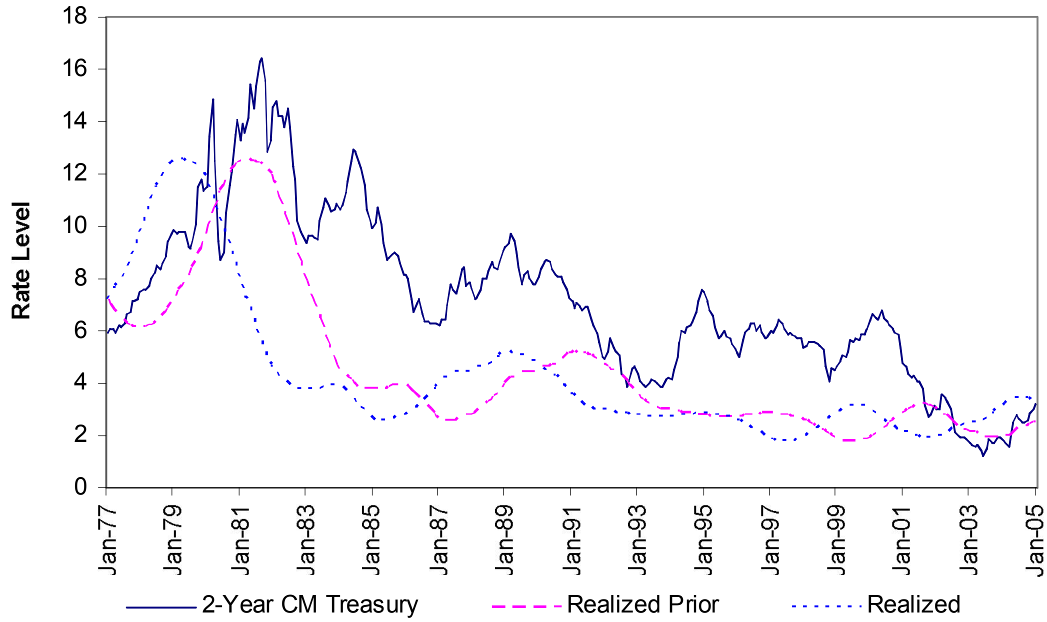
EXHIBIT 1
Summary of Comparison Between Fixed Income and Volatility as Asset Classes

Comparison Item	Fixed Income	Volatility
Position	Long bonds	Short volatility: short options / short variance swaps / short volatility swaps
Driver of Risk Premium	Investors receive a risk premium for a) providing governments and other entities with needed capital, b) taking risk that inflation will exceed the interest received from the bond, and c) taking interest rate risk (the term premium) associated with the volatility of market interest rates	Investors receive a risk premium for a) selling insurance against a market crisis b) taking risk that realized vol will exceed implied vol, thus rendering the position unprofitable, and c) taking ongoing mark-to-market risk associated with the volatility of implied volatility
Market Quotes*	The standard for quoting prices for bonds is the interest rate (or yield to maturity), which is then used in a present value formula in order to compute the actual price paid for the security	The standard for quoting prices for volatility instruments is the instrument's implied volatility, which is then used in an options pricing model (e.g., the Black-Scholes model) to compute the price paid for the instrument or the relevant swap rate
Summary Nature of Price Quote*	The yield to maturity is the average interest rate that, if used to discount all cash flows, will make the bond price equal to the summed value of those discounted cash flows	The implied volatility is the average volatility over the life of an option that will make the expected present value of the option's replicating portfolio equal to the options current price
Mean Reverting Property	It is widely accepted that interest rates are a mean reverting process	It is widely accepted that implied volatility is a mean reverting process
Term Structure	Bonds are offered at various maturities, thus forming what is typically an upward sloping term structure	Options and other volatility products are offered at various expiries / maturities, thus forming what is typically an upward sloping term structure

*These items were included in a similar table in a presentation written by Emanuel Derman (Derman [2003]).

EXHIBIT 2
Risk Premia Offered by Fixed Income and Volatility

Panel 1: 2-Year Constant Maturity Treasury Compared to Average Prior Inflation and Average Actual Inflation



Panel 2: VIX Implied Volatility Compared to Prior Realized Volatility and Actual Realized Volatility

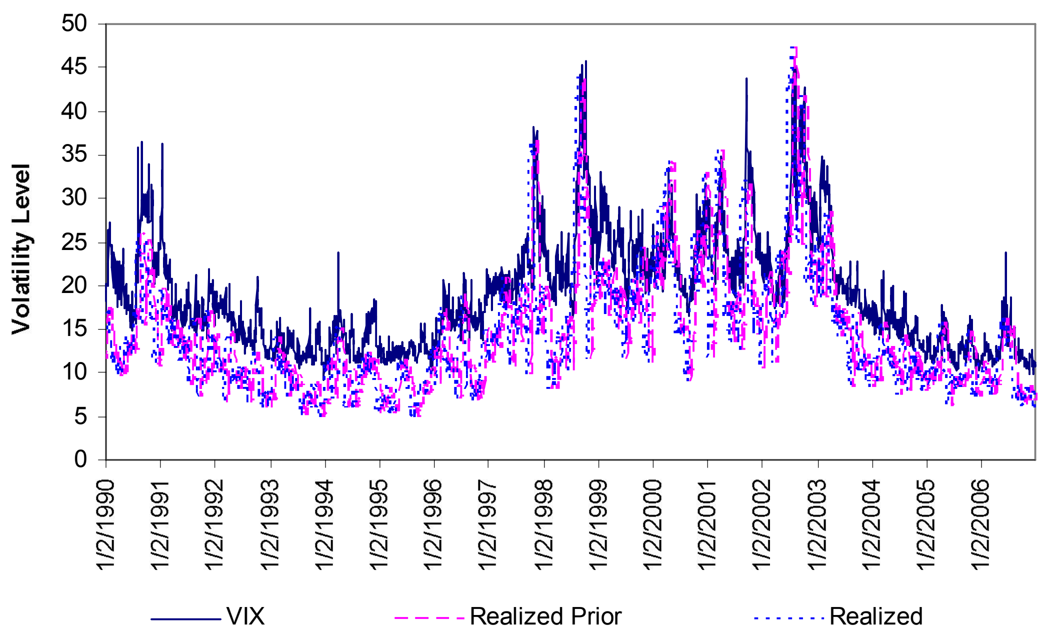


EXHIBIT 3
Risk Premia Offered by Fixed Income and Volatility—Summary of Data

Panel 1: Risk Premium of 2-Year Constant Maturity Treasury Over Prior 24-Month Average Inflation and Over Actual 24-Month Average Inflation

	2-Yr Treasury Yield	Inflation: Prior Realized	Difference: 2Yr Tsy-Prior	Inflation: Actual Realized	Difference: 2Yr Tsy-Actual
All Data	7.16	4.54	**2.62**	4.30	**2.86**
1/31/1977 to 12/31/1990	9.59	6.24	**3.35**	5.98	**3.61**
1/31/1991 to 1/31/2005	4.74	2.85	**1.89**	2.63	**2.12**

Panel 2: VIX Volatility Index as Compared to Prior 21-Day Realized Volatility and as Compared to Actual Realized 21-Day Volatility

	VIX Level (Implied Vol)	Prior Realized Volatility	Difference: VIX-Prior Realized	Actual Realized Volatility	Difference: VIX-Actual Realized
All Data	19.06	14.35	**4.71**	14.31	**4.75**
1/2/1990 to 6/30/1998	17.11	11.76	**5.35**	11.75	**5.36**
7/1/1998 to 12/28/2006	21.02	16.95	**4.07**	14.31	**6.71**

EXHIBIT 4
Mean Reversion Parameters and Related T-Statistics

Panel 1: Constant Maturity 2-Year Treasury Rate—Mean Reversion Parameters and Related T-Statistics

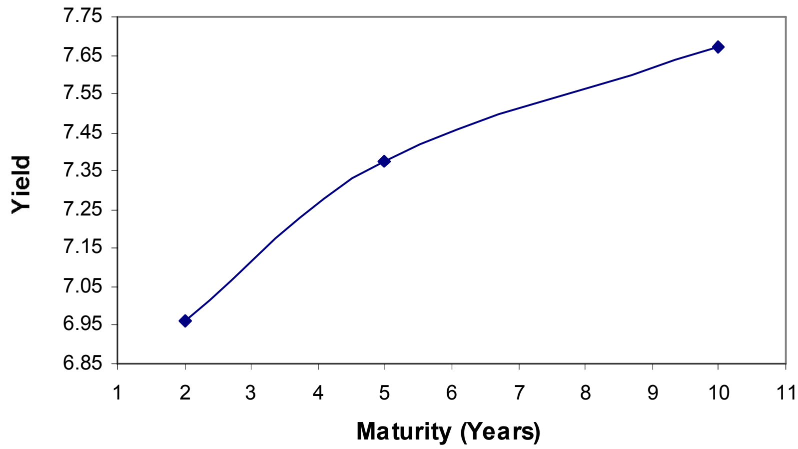
	Mean Reversion Parameter	T-Statistic
All Data	0.0093	120.4
1/31/1977 to 12/31/1990	0.0364	49.4
1/31/1991 to 1/31/2005	0.0169	78.7

Panel 2: VIX Volatility Index—Mean Reversion Parameters and Related T-Statistics

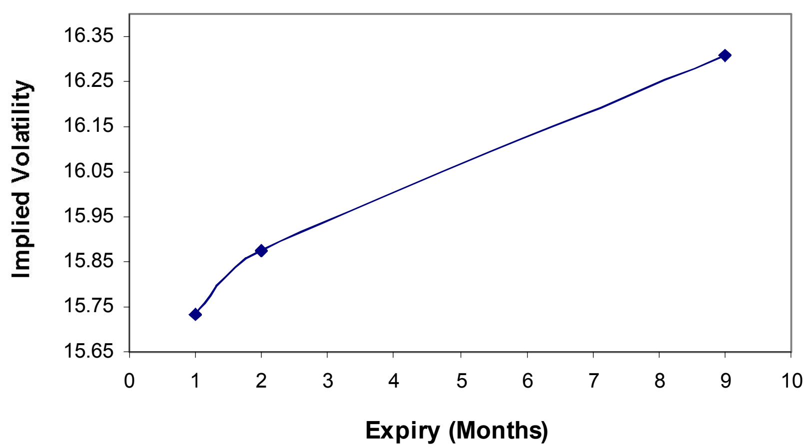
	Mean Reversion Parameter	T-Statistic
All Data	0.0172	346.5
1/2/1990 to 6/30/1998	0.0244	205.3
7/1/1998 to 12/28/2006	0.0163	279.8

EXHIBIT 5
Fixed Income and Volatility Term Structures

Panel 1: U.S. Treasury Term Structure (Monthly Average 1/31/77 to 1/31/07)



Panel 2: Volatility Term Structure (Daily Average 6/03/02 to 1/31/07)



The term structure data for interest rates consists of on-the-run Treasuries of the given maturity (Bloomberg series GT2 Govt, GT5 Govt, and GT10 Govt). The volatility data is the 50 delta-point data for S&P 500 options (SPX) as provided by Bloomberg. The leap month is the first option with an expiry of nine months or longer. The same data is used in Exhibits 6 and 7.

EXHIBIT 6
Volatility of Interest Rates and Volatilities Across the Term Structure

Panel 1: Volatility of Interest Rates at Different Points of the Term Structure (Monthly 1/31/77 to 1/31/07)

	2-Year	5-Year	10-Year
Standard Deviation of Interest Rates	3.20	2.95	2.76

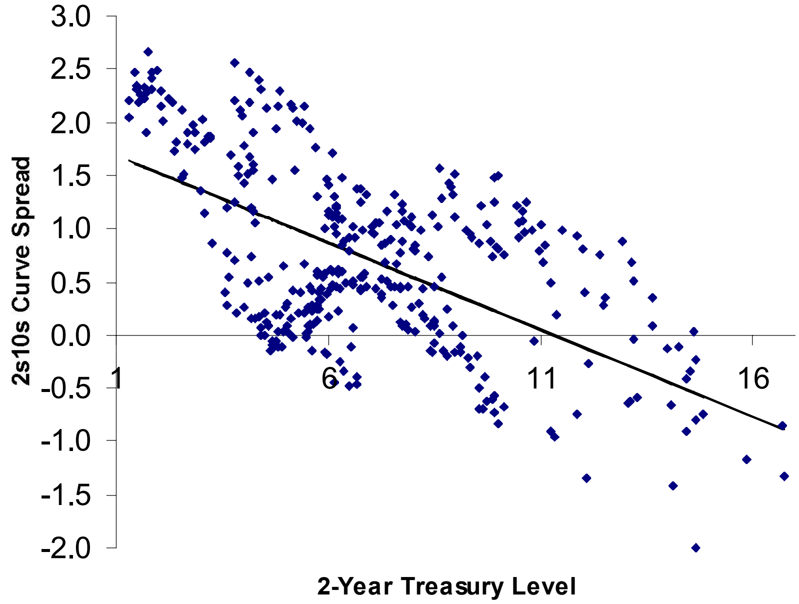
Panel 2: Volatility of Implied Volatility at Different Points of the Term Structure (Daily 6/03/02 to 1/31/07)

	1-Month	2-Month	Leap-Month
Standard Deviation of Implied Volatility	7.12	6.15	5.82

These are the volatilities of the interest rate and 50 delta-point time series using data as described in Exhibit 5.

EXHIBIT 7
Relationship Between Level and Curve Spread

Panel 1: Relationship Between the Level of the 2-Year Treasury Yield and the 2s10s Curve Spread (Monthly 1/31/77 to 1/31/07)



Panel 2: Relationship Between the Level of the VIX Volatility Index and the 1-Curve / Leap-Month Term Spread (Daily 6/03/02 to 1/31/07)

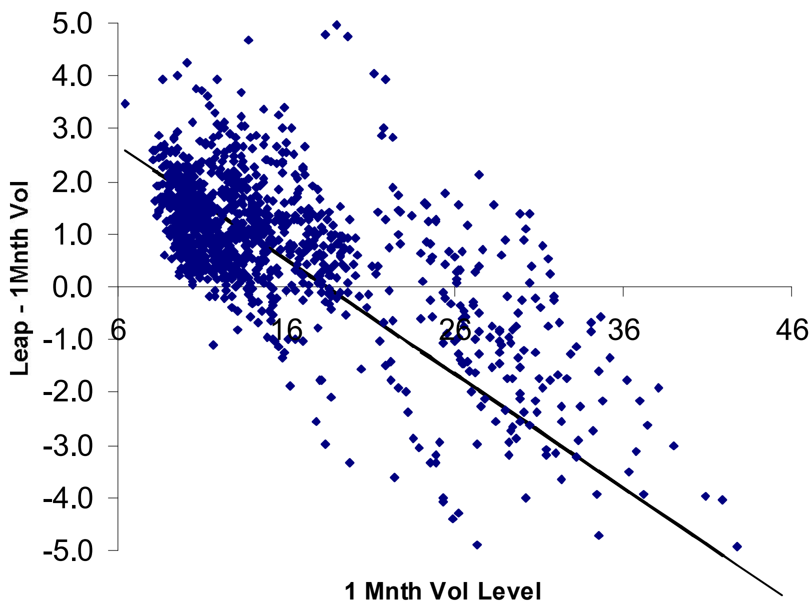


EXHIBIT 8
Summary of Strategies for Capturing the Volatility Risk Premium

Strategy	Implementation	Characteristics
Covered Calls	Sell call options (typically at-the-money or slightly out-of-the-money) on an underlying already held as a long position in the investor's portfolio	Allows the investor to capture the volatility risk premium, but at the cost of an altered return distribution of the underlying portfolio—large gains will be truncated by the short call positions
Covered Puts	Sell put options (typically at-the-money or slightly out-of-the-money) on an underlying already held as a short position in the investor's portfolio	Again, allows capture of the volatility risk premium, but provides for reduced downside protection in a sell-off, as the short put positions take out the short positions in the underlying
Delta-Hedged Options	Sell at-the-money options, hedge the delta exposure using the underlying; rebalance daily or more frequently depending on the size of the move	Provides for volatility exposure without direct market exposure. Generally gains if the implied volatility paid for the options is more than the actual realized volatility of the underlying. Requires ongoing attention to in order to rehedge
Short Variance (Volatility) Swap	Receive fixed in a variance (volatility) swap (a rather simple process)	Pays the difference between the variance (volatility) strike and realized variance (volatility). From the client's perspective, it requires no rehedging
Short VIX Futures	Sell VIX futures	One profits if future implied volatility is lower than current expectations for future implied volatility; this position does not capture a risk premium in the sense described in this paper
Short Corridor Variance Swaps, Conditional Variance Swaps, or Gamma Swaps	Receive fixed in one of these contracts (again, a rather simple process)	These more exotic versions of variance swaps can be used to 1) reduce the variance strike level, 2) express views on both volatility and the level of the underlying simultaneously, and 3) express views on the shape of the volatility skew